KTH ROYAL INSTITUTE
OF TECHNOLOGY

Optimal Order Strategies on the Day-Ahead Electricity Market

Martin Biel

20/9-2017

Outline

Outline

Background

Problem Description

Contribution

Optimal Strategies

Outlook on Future Work

Background - Motivation

- Simulation of hydro power operations $\rightarrow$ Decision-support

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  - Price forecasts
  - Irregular power production: solar and wind
  - Nuclear power phase-out

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- Simulation of hydro power operations $\rightarrow$ Decision-support
  - Price forecasts
  - Irregular power production: solar and wind
  - Nuclear power phase-out
- Common: Trade-off between accuracy and computation time

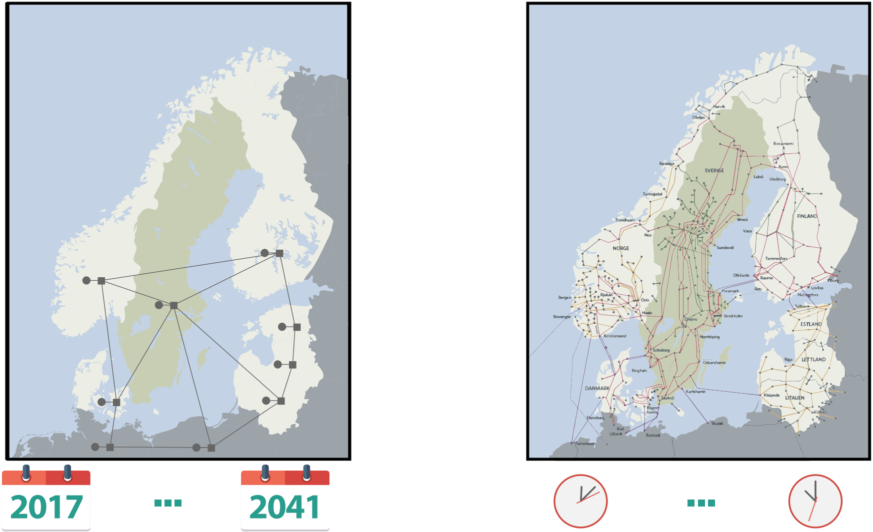

Figure: Simulations of hydro power operations

Background - Aim

Provide reliable decision-support in real time

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- Accurate models
- Fast computations

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Provide reliable decision-support in real time

- Accurate models
  - Optimal model reductions
- Fast computations
  - Scalable algorithms that make efficient use of commodity hardware

Backgound - Approach

Stochastic programming for hydro power operations:

- Optimal orders on the day-ahead market
- Maintenance scheduling
- Long-term investments
- Wind/solar uncertainties

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Improvements

- Multiple scenarios $\rightarrow$ More accurate models
- Parallel decomposition $\rightarrow$ Faster computations

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- **Optimal orders on the day-ahead market**
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Problem Description - Scandinavian Electricity Market

Electricity markets in Scandinavia deregulated since 90s

- Norway 1991
- Sweden 1996
- Finland 1998
- Denmark 2000

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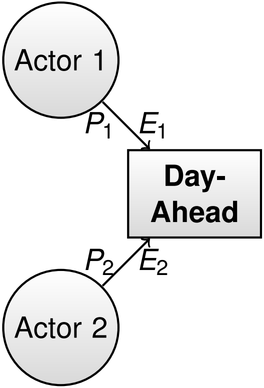
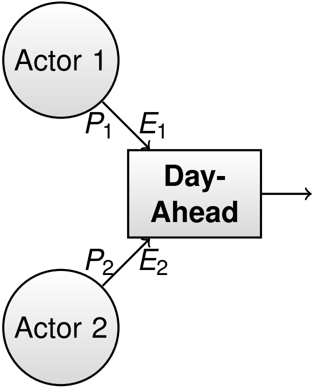
Energy volumes actively traded on a competitive market: Nord Pool

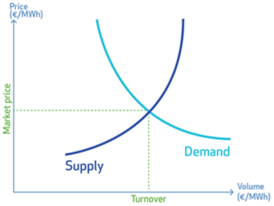
- Day-ahead market
- Intraday market

Problem Description - Electricity Market

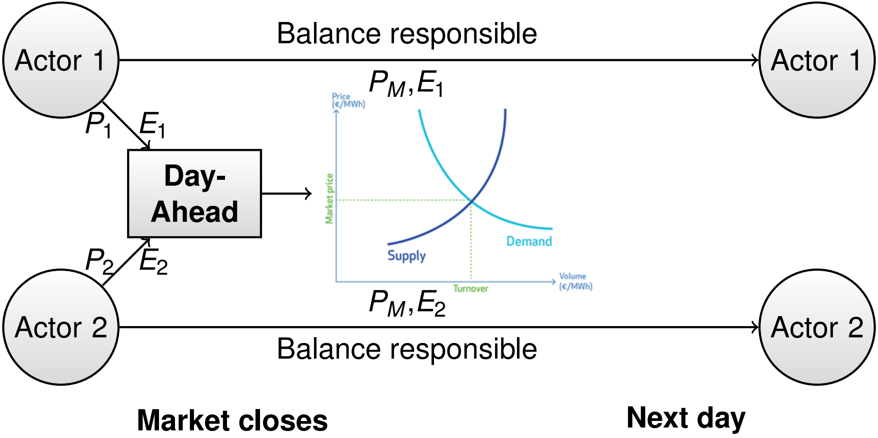


Problem Description - Electricity Market



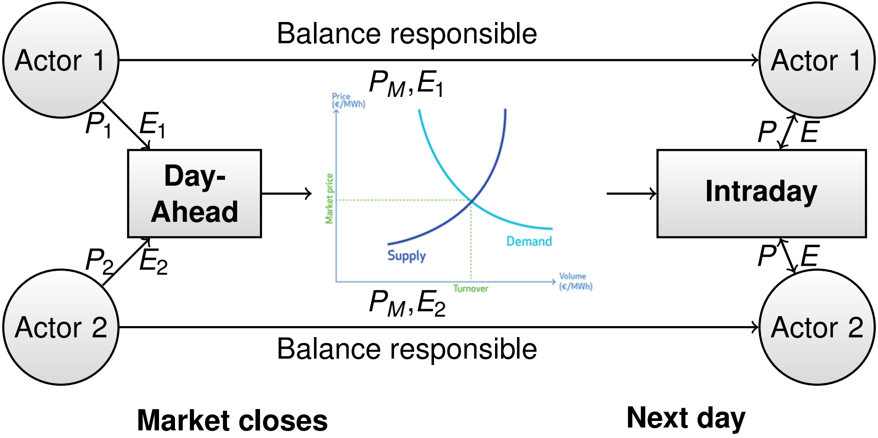


Market closes



Balance responsible
Actor 1
Actor 1
P_M, E_1
P_1
E_1
Day-Ahead
Price (€/MWh)
Market price
Supply
Demand
Volume (€/MWh)
Turnover
P_2
E_2
P_M, E_2
Actor 2
Actor 2
Balance responsible
Market closes
Next day

Problem Description - Electricity Market



Problem Description - The Day-Ahead Market

Order Types

- Single Hourly Order
  - Price independent
  - Price Dependent
- Block Order
  - Regular
  - Linked
- Exclusive Group
- Flexible Order

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Problem Description - Single Order

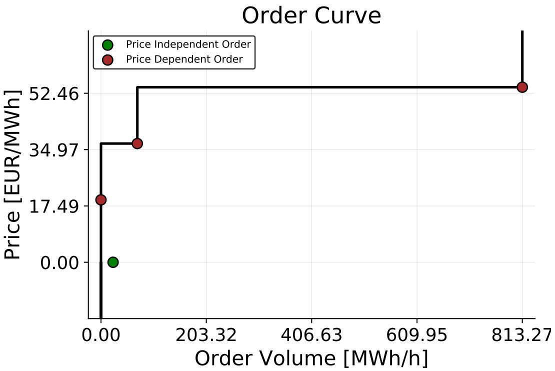

Figure: Single hourly order

Problem Description - Single Order

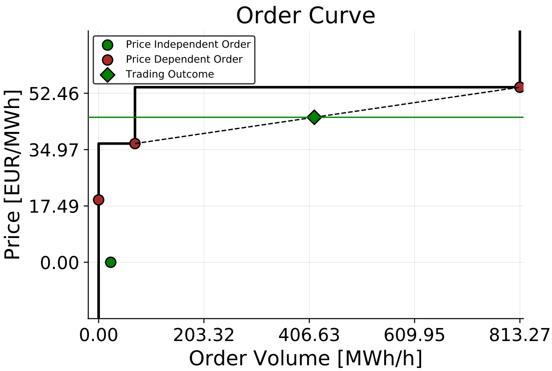

Figure: Interpolated energy volume for a given market price

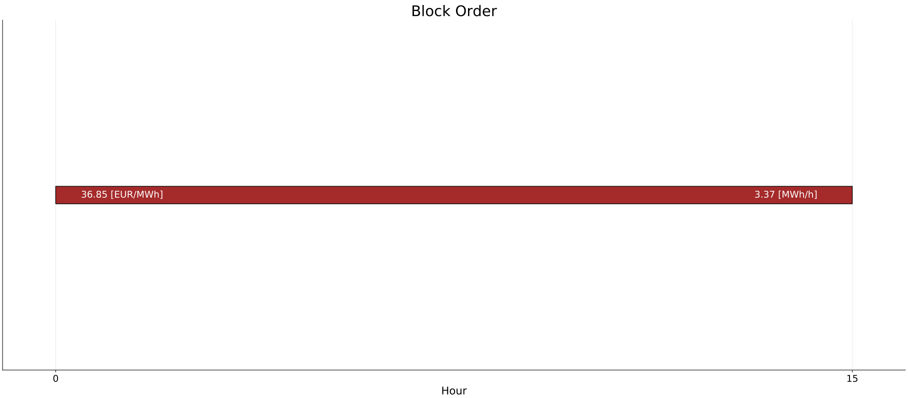

Figure: Block order between 00:00-15:00

Figure: Rejected after market price settlement

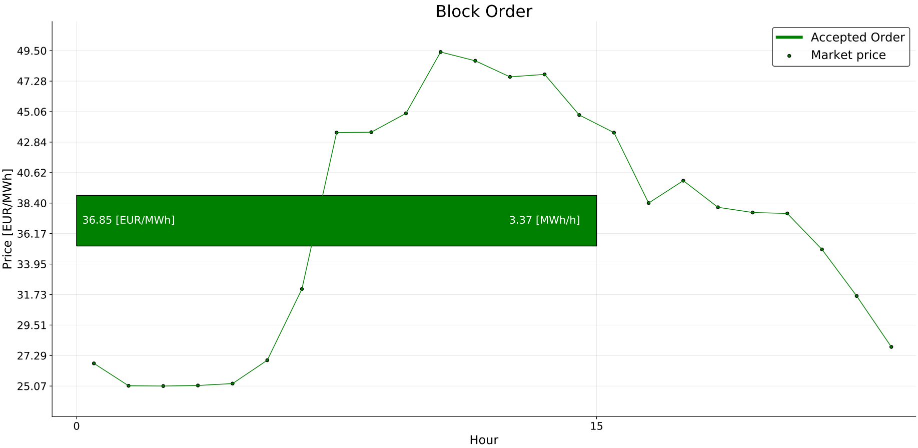

Figure: Accepted after market price settlement

Problem Description - Optimal Order Strategies

- Optimal orders given price forecasts
- Price taking hydro power producer
- Next day production governed by hydroelectric model

Outline

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Contribution - Model Framework

Data

- Physical data of Swedish hydro power stations
  - Topologies
  - Capacitites
  - Flow times
- Financial data from Nord Pool
  - historic prices

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- Physical data of Swedish hydro power stations
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Julia: JuMP + StructJuMP

- Domain-specific modeling language for mathematical optimization
- Optimization models can be processed programatically

HydroModels.jl

```
# Variables
# ==========================================================
@variable(internalmodel,xt_d[i = model.bids, t = model.hours] >= 0)
@variable(internalmodel,yt[t = model.hours] >= 0)
@variable(internalmodel,H[t = model.hours] >= 0)

# Define objective
# ==========================================================
@objective(internalmodel, Max, net_profit + value_of_stored_water)

# Constraints
# ==========================================================
# Load balance
@constraint(internalmodel,loadbalance[s = model.scenarios, t = model.hours],
            yt[s,t] + sum(yb[s,b]
                          for b = find(A->in(t,A),model.hours_per_block))
           - H[s,t] == z_up[s,t] - z_do[s,t]
            )
```

HydroModels.jl

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            )
```

+

Byarforsen	134	134	17	200	0	0	124	75	75	Ljusnan
Krokstrommen	135	135	100	200	0	830	130	30	30	Ljusnan
Langstrommen	136	136	48	180	0	278	131	25	40	Ljusnan
Storastrommen	137	137	24	180	0	556	142	35	35	Ljusnan
Ojeforsen	138	138	25	190	0	278	143	55	55	Ljusnan
Laforsen	139	139	57	190	0	830	149	280	280	Ljusnan

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=

```
data = HydroModelData("data/plantdata.csv","data/spotpricedata.csv")
dayahead = DayAheadModel(data,5,"Ljusnan")
plan!(dayahead)
```

Outline

Contribution - Stochastic Day-Ahead Model

$$\begin{aligned}\text{maximize} \quad & \text{Profit} + \text{Water Value} - \text{Balance Cost} \\ \text{subject to} \quad & \text{Day-Ahead Orders} \\ & \text{Physical Limitations} \\ & \text{Economic/Legal Limitations}\end{aligned}$$

Contribution - Stochastic Day-Ahead Model

Day-Ahead Orders - $\mathbf{x} \in \mathcal{X}$

- Indices $t \in T := \{1, \ldots, 24\}, b \in B := \{b = (t_a, \ldots, t_b) : t_i \in T\}$
- Price independent: $x^i_t \geq 0$
- Price dependent: $0 \leq x^d_{i,t} \leq x^d_{i,t+1}$
- Block: $x^b_{j,b} \geq 0$

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- Price dependent: $0 \leq x_{i,t}^d \leq x_{i,t+1}^d$
- Block: $x_{j,b}^b \geq 0$

Scenario Outcomes - $\mathbf{y} \in \mathcal{Y}(\mathbf{x}, \xi)$

$$y_t = x_t^i + \frac{\rho_t^\xi - p_i}{p_{i+1} - p_i} x_{i+1,t}^d + \frac{p_{i+1} - \rho_t^\xi}{p_{i+1} - p_i} x_{i,t}^d, \quad p_i \leq \rho_t^\xi \leq p_{i+1}$$

$$y_b = \sum_{j=1}^{\bar{j}(b)} x_{j,b}, \quad \bar{j}(b) = \max \left\{ i : p_i \leq \frac{1}{|b|} \sum_{t \in b} \rho_t^\xi \right\}$$

Contribution - Stochastic Day-Ahead Model

Next Day Production - $\mathbf{h} \in \mathcal{H}(\mathbf{y})$

- Indices $p \in P := \{$All power stations operable by the producer$\}$
- Water discharge/spillage: $0 \leq Q_{p,t} \leq \overline{Q}_p, S_{p,t} \geq 0$
- Reservoir content: $0 \leq M_{p,t} \leq \overline{M}_p$
- Energy production: $H_{p,t} \geq 0$
- Local inflow/outflow: V_p
- Power imbalances: $z_t^+, z_t^- \geq 0$

Contribution - Stochastic Day-Ahead Model

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Load Balance

$$L(\mathbf{y}, \mathbf{h}) : y_t + \sum_{b \in B: t \in b} y_b - \sum_p H_t = z_t^+ - z_t^-$$

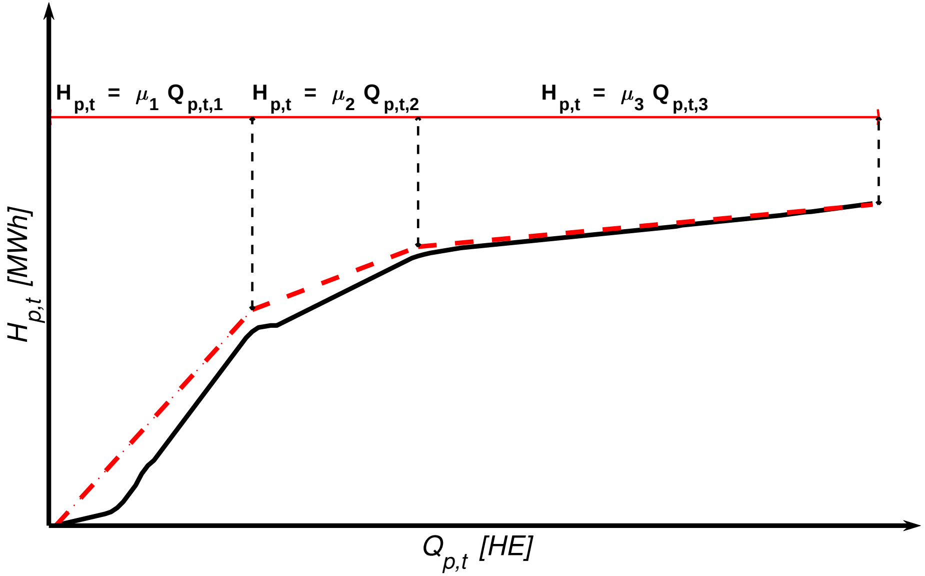
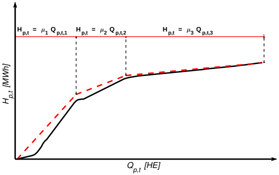


Figure: Piecewise linear production equivalent

Contribution - Stochastic Day-Ahead Model

Hydro power production



$$\rightarrow H_{p,t} = \sum_{s} \mu_{p,s} Q_{p,t,s}$$

Figure: Piecewise linear production equivalent

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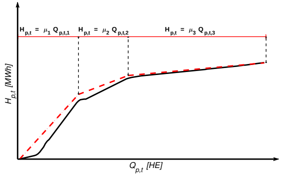
Hydro power production



$$\rightarrow H_{p,t} = \sum_{s} \mu_{p,s} Q_{p,t,s}$$

Figure: Piecewise linear production equivalent

Hydrological balance

$$M_{p,t} = M_{p,t-1} - Q_{p,t} - S_{p,t} + \sum_{p_q \in Q_u} Q_{p_q,t-\tau_{p_q}} + \sum_{p_s \in S_u} S_{p_s,t-\tau_{p_s}} + V_p$$

Contribution - Stochastic Day-Ahead Model

Water value

$$W(\mathbf{h}) = \lambda_f \sum_{p} M_{p,24} \sum_{p_q \in Q_d, s} \mu_{p_q,s}$$

Contribution - Stochastic Day-Ahead Model

Water value

$$W(\mathbf{h}) = \lambda_f \sum_p M_{p,24} \sum_{p_q \in Q_d, s} \mu_{p_q,s}$$

Profit

$$\Pi(\mathbf{y}) = \sum_t \rho_t^\xi y_t + \sum_b |b| \bar{\rho}_b^\xi y_b - \sum_t (\lambda_t^+ z_t^+ - \lambda_t^- z_t^-)$$

Contribution - Stochastic Day-Ahead Model

Water value

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$$\Pi(\mathbf{y}) = \sum_t \rho_t^\xi y_t + \sum_b |b| \bar{\rho}_b^\xi y_b - \sum_t (\lambda_t^+ z_t^+ - \lambda_t^- z_t^-)$$

Objective

$$Q(\mathbf{y}, \mathbf{h}, \xi) = W(\mathbf{h}) + \Pi(\mathbf{y})$$

Contribution - Stochastic Day-Ahead Model

Complete Model

$$
\begin{aligned}
\min\ & \mathbb{E}_\xi \left[Q(\mathbf{y}, \mathbf{h}, \xi) \right] \\
\text{s.t. } & \mathbf{x} \in \mathcal{X} \\
& \mathbf{y} \in \mathcal{Y}(\mathbf{x}, \xi) \\
& \mathbf{h} \in \mathcal{H}(\mathbf{y}) \\
& L(\mathbf{y}, \mathbf{h}) = 0
\end{aligned}
$$

Outline

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Problem Description

Contribution

- Model Framework
- Stochastic Day-Ahead Model
- Optimization Algorithms

Optimal Strategies

Outlook on Future Work

Contribution - Optimization Algorithms

Benders decomposition for stochastic programming

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Benders decomposition for stochastic programming

- L-Shaped [Van Slyke,Wets]

Contribution - Optimization Algorithms

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- Trust-Region L-Shaped [Linderoth,Wright]

Contribution - Optimization Algorithms

Benders decomposition for stochastic programming

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- Regularized Decomposition [Ruszczyński]
- Trust-Region L-Shaped [Linderoth,Wright]

LShaped.jl

```
ls = LShapedSolver(model,x0)
rls = RegularizedLShapedSolver(model,x0)
trls = TrustRegionLShapedSolver(model,x0)
```

Contribution - Parallel Optimization Algorithms

The algorithms are cutting-plane methods:

- Solve subproblems and generate cutting-planes
- Update and resolve a master problem

$$
\begin{array}{llcll}
\min & c^T x + \mathbb{E}_\xi \left[\min\limits_{y \in \mathcal{Y}(x)} Q(y, \xi) \right] & & \min & c^T x + \sum\limits_{i=1}^{n} \theta_i \\
\text{s.t.} & x \in \mathcal{X} & \rightarrow & \text{s.t.} & x \in \mathcal{X} \\
 & & & & \partial Q_i x + \theta_i = q_i
\end{array}
$$

Contribution - Parallel Optimization Algorithms

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$$\begin{array}{ll} \min & c^T x + \mathbb{E}_\xi \left[\min\limits_{y \in \mathcal{Y}(x)} Q(y, \xi) \right] \\ \text{s.t.} & x \in \mathcal{X} \end{array} \quad \rightarrow \quad \begin{array}{ll} \min & c^T x + \sum\limits_{i=1}^{n} \theta_i \\ \text{s.t.} & x \in \mathcal{X} \\ & \partial Q_i x + \theta_i = q_i \end{array}$$

Readily extendable to asynchronous variants

- Master problem is solved on a master node
- Subproblems are distributed among workers

Contribution - Parallel Optimization Algorithms

Idea to exploit structure and make use of commodity hardware

Contribution - Parallel Optimization Algorithms

Idea to exploit structure and make use of commodity hardware

- Linear subproblems have the same underlying matrix structure

Contribution - Parallel Optimization Algorithms

Idea to exploit structure and make use of commodity hardware

- Linear subproblems have the same underlying matrix structure
  - LU factorize once and store on GPU
  - Reuse for efficient linear solves during simplex iterations

Contribution - Summary

- HydroModels.jl
  - Possible to extend to other models of hydro power operations
- Day-Ahead Model
  - Optimization formulation
  - Visualization tools
- LShaped.jl
  - 3 fully implemented serial L-Shaped variants
  - 1 parallel implementation (work in progress)

Contribution - Summary

- HydroModels.jl
  - Possible to extend to other models of hydro power operations
  - Modular
- Day-Ahead Model
  - Optimization formulation
  - Visualization tools
- LShaped.jl
  - 3 fully implemented serial L-Shaped variants
  - 1 parallel implementation (work in progress)
  - Modular

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Figure: Courtesy of VRF (http://www.vattenreglering.se/)

Example 1: Ljusnan

- 21 power stations
- 5 price curves from historic data

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- 21 power stations
- 5 price curves from historic data

Day-Ahead model with:

- 9305 linear constraints
- 18274 variables

Example 1: Single Order

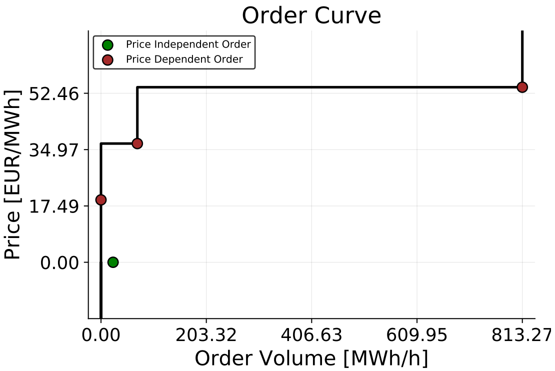


Figure: Single order during the first hour

Figure: All single orders in optimal strategy

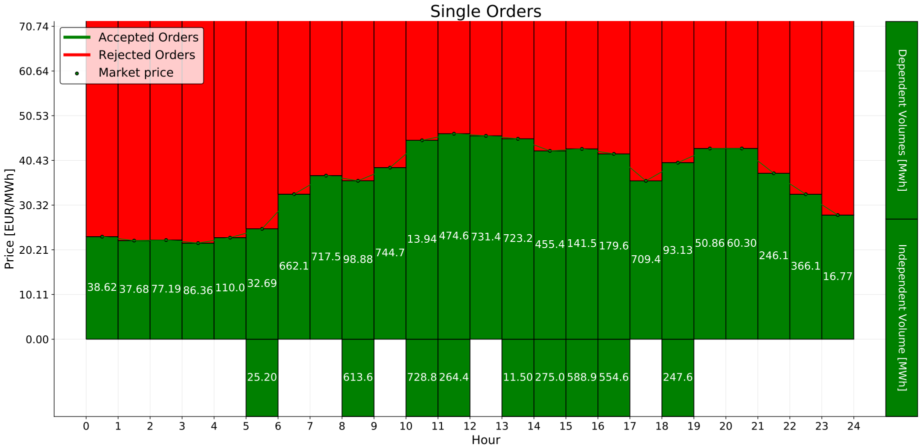


Figure: Single order outcome of optimal strategy, for a given price curve

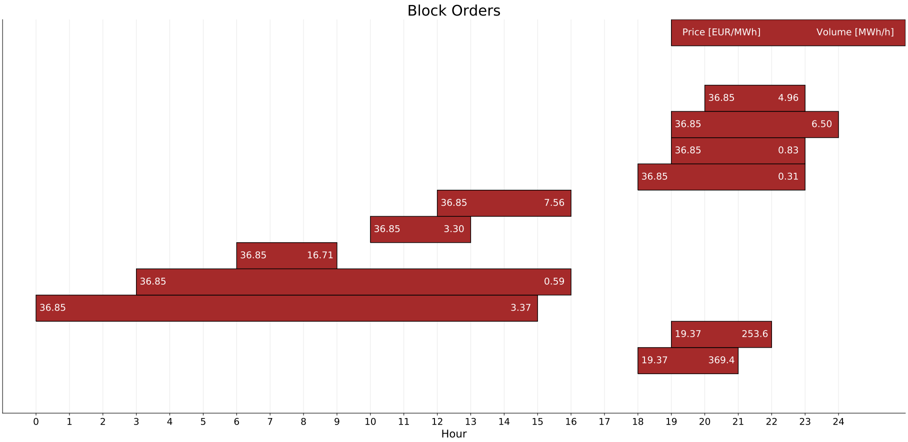
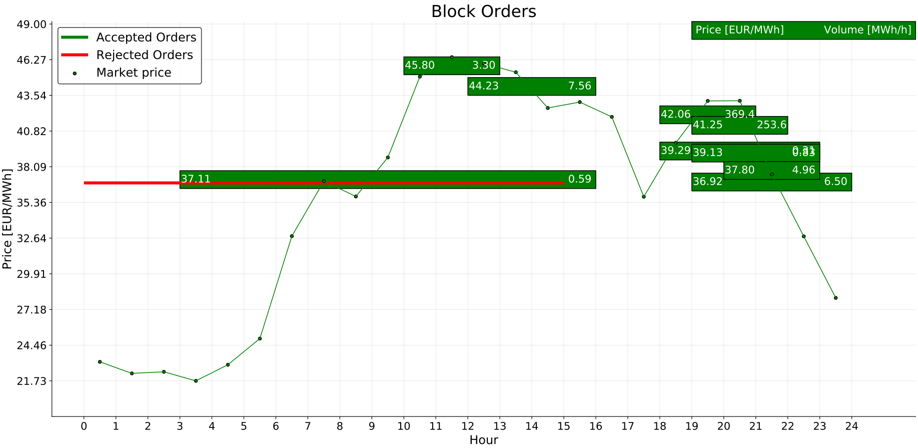


Figure: All block orders in optimal strategy



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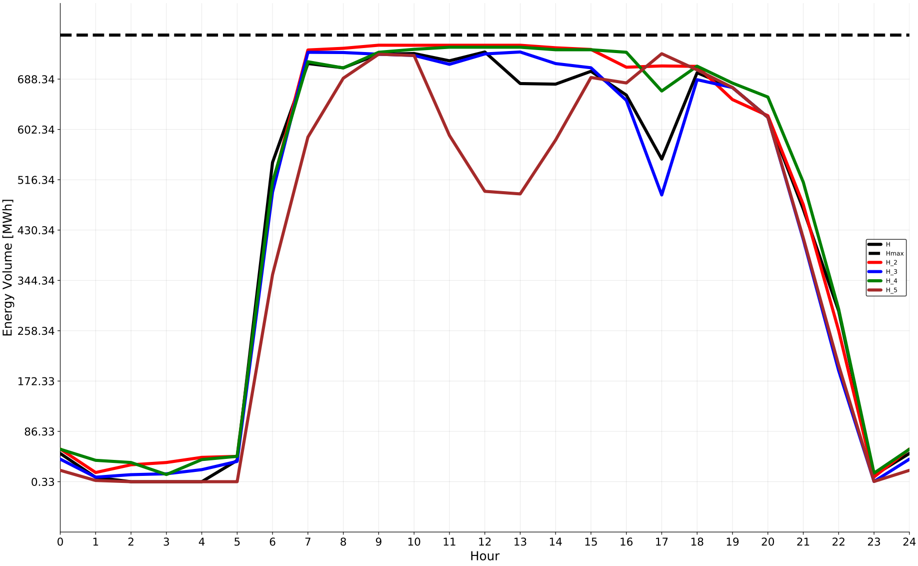

Figure: Energy production in all scenarios

Outline

Example 2: All rivers

- 257 power stations
- 20 price curves from historic data

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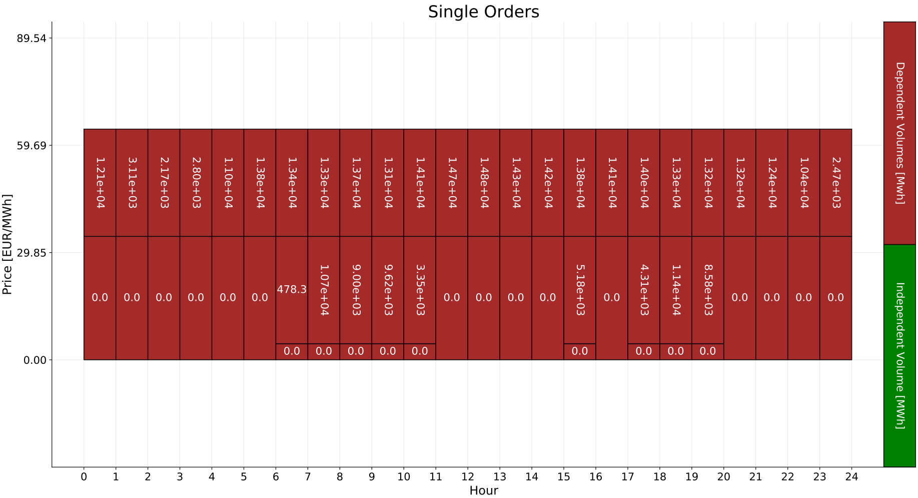
Day-Ahead model with:

- 376700 linear constraints
- 748043 variables



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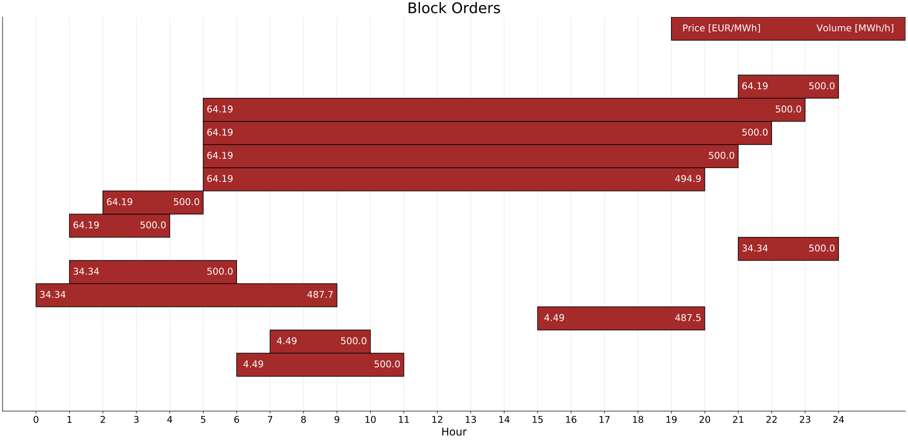

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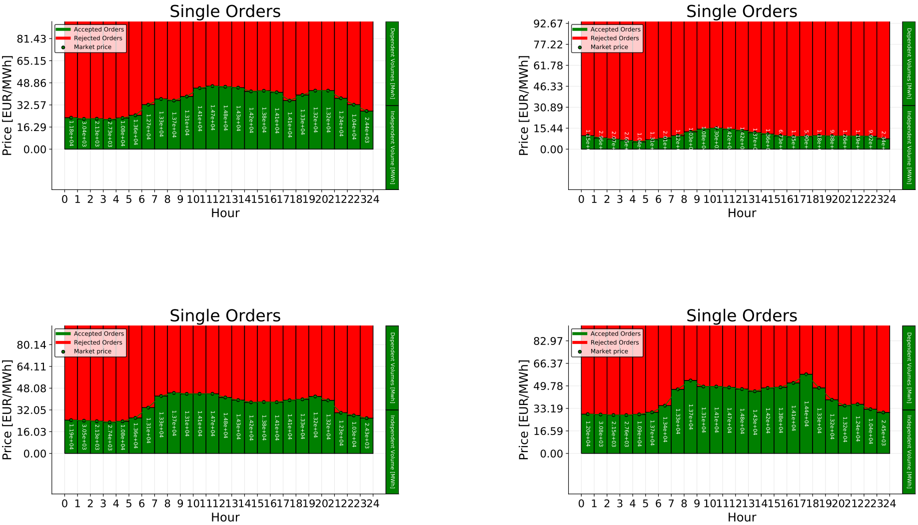

Figure: Single order outcomes of optimal strategy, for 4 different price curves

Figure: Block order outcomes of optimal strategy, for two given price curves

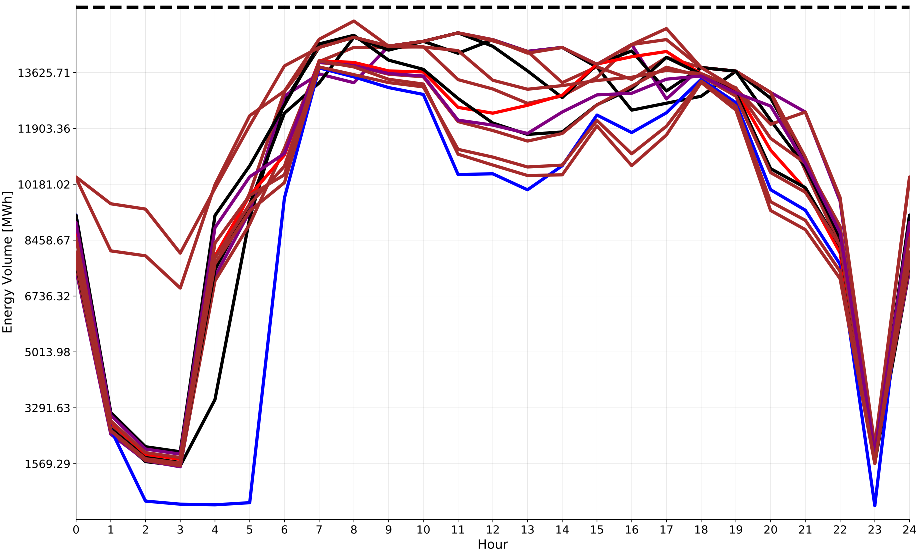

Figure: Energy production in all scenarios

Outline

Future Work

Model framework

Day-Ahead Model

Optimization Algorithms

Future Work

Model framework

- Implement more models of hydro power operations

Day-Ahead Model

Optimization Algorithms

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Day-Ahead Model

- Generate price curves from statistic model
- Allow varying order prices

Optimization Algorithms

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Optimization Algorithms

- Finish parallel variants
- Evaluate on day-ahead model