Stochastic Programming for Hydropower Operations

Modeling and Algorithms

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Motivation

- Simulation of hydro power operations $\rightarrow$ Decision-support

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  - Price forecasts
  - Irregular power production: solar and wind
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  - Fast computations: Scalable algorithms on commodity hardware

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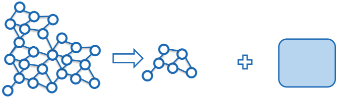

Manageable models

Scalable algorithms

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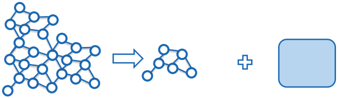

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Stochastic programming for hydro power operations

- Optimal orders on the day-ahead market
- Maintenance scheduling
- Long-term investments
- Wind/solar uncertainties

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- Long-term investments
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Advantages

- Multiple scenarios $\rightarrow$ More accurate models
- Parallel decomposition $\rightarrow$ Faster computations

Contribution

Julia modules

- StochasticPrograms.jl
- LShapedSolvers.jl
- HydroModels.jl

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Software Innovations

- Deferred model creation
- Data injection

Content

- Initial approach

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- Final remarks

Initial Approach

- HydroModel
  - Data
  - JuMP model

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- Julia struct for each model: ShortTerm, DayAhead

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  - JuMP model
- Julia struct for each model: ShortTerm, DayAhead
- Parallel decomposition: L-shaped on StructJuMP.jl models
- Performance: Solve extended form using MathProgBase solvers

Initial Approach

```
function define_structjump_problem(model::DayAheadModel)
    model.internalmodels[:structured] = StructuredModel(num_scenarios = numscenarios(model))
    params = model.modeldata
    ..
    @variable(internalmodel,xt_i[t = model.hours] >= 0)
    ..
    for s in 1:numscenarios(model)
        block = StructuredModel(parent = internalmodel, id = s)
        ...
        @variable(block,Q[p = model.plants, q = model.segments, t = model.hours],
                  lowerbound = 0,upperbound = params.Q̄[(p,q)])
        @variable(block,S[p = model.plants, t = model.hours] >= 0)
        ...
        @expression(block,value_of_stored_water,
                    params.λ_f*sum(M[p,hours(model.horizon)]*sum(params.μ[i,1]
                        for i = params.Qd[p])
                            for p = model.plants))
        # Define objective
        @objective(block, Max, net_profit + value_of_stored_water)
        ...
        @constraint(block,production[s = model.scenarios, t = model.hours],
                    H[s,t] == sum(params.μ[p,q]*Q[s,p,q,t]
                                  for p = model.plants, q = model.segments)
                    )
        ...
    end
end
```

Initial Approach

```
function define_dep_problem(model::DayAheadModel)
    model.internalmodels[:dep] = Model()
    params = model.modeldata
    ...
    @variable(internalmodel,xt_i[t = model.hours] >= 0)
    ...
    @variable(block,Q[s = model.scenarios, p = model.plants, t = model.hours],
          lowerbound = 0, upperbound = params.Q̄[(p,q)])
    @variable(block,S[s = model.scenarios, p = model.plants, t = model.hours] >= 0)
    ...
    @expression(block,value_of_stored_water,
        sum(scenarios[s].π*params.λ_f*sum(M[s,p]*sum(params.μ[i,1]
                                          for i = params.Qd[p])
                                              for p = model.plants)
                                                  for s = model.scenarios))
    # Define objective
    @objective(block, Max, net_profit + value_of_stored_water)
    ...
    @constraint(block,production[s = model.scenarios, t = model.hours],
        H[s,t] == sum(params.μ[p,q]*Q[s,p,q,t]
                    for p = model.plants, q = model.segments)
        )
        ...
end
```

Initial Approach - Issues

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- ...but StructJuMP relies on MPI

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- A lot of code repetition
- No clearcut way to calculate stochastic measures: EVPI, VSS
- The model creation is somewhat inflexible
- Parallel L-shaped using the Distributed module in Julia...
- ...but StructJuMP relies on MPI
- Creating a new hydromodel involves reimplementing a new type

New Approach

- StochasticPrograms.jl

- HydroModels.jl

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  - Model creation focused on data and optimization formulation
  - Efficient model reinitialization

New Approach

- StochasticPrograms.jl
  - Flexible model creation
  - Parallel capabilities based on the Distributed module
  - Stochastic programming constructs
- HydroModels.jl
  - Model creation focused on data and optimization formulation
  - Efficient model reinitialization
  - Predefined models
    - Short-term production planning
    - Optimal orders on the day-ahead market

StochasticPrograms.jl - Simple Example

$$
\begin{aligned}
\underset{x_1, x_2 \in \mathbb{R}}{\text{minimize}} \quad & 100x_1 + 150x_2 + \mathbb{E}_\omega[Q(x_1, x_2, \xi)] \\
\text{s.t.} \quad & x_1 + x_2 \leq 120 \\
& x_1 \geq 40 \\
& x_2 \geq 20
\end{aligned}
$$

where

$$
\begin{aligned}
Q(x_1, x_2, \xi) = \min_{y_1, y_2 \in \mathbb{R}} \quad & q_1(\xi)y_1 + q_2(\xi)y_2 \\
\text{s.t.} \quad & 6y_1 + 10y_2 \leq 60x_1 \\
& 8y_1 + 5y_2 \leq 80x_2 \\
& 0 \leq y_1 \leq d_1(\xi) \\
& 0 \leq y_2 \leq d_2(\xi)
\end{aligned}
$$

StochasticPrograms.jl - Simple Example

```julia
sp = StochasticProgram(solver=ClpSolver())

@first_stage sp = begin
    @variable(model, x₁ >= 40)
    @variable(model, x₂ >= 20)
    @objective(model, Min, 100*x₁ + 150*x₂)
    @constraint(model, x₁+x₂ <= 120)
end

@second_stage sp = begin
    @decision x₁ x₂
    s = scenario
    @variable(model, 0 <= y₁ <= s.d[1])
    @variable(model, 0 <= y₂ <= s.d[2])
    @objective(model, Min, s.q[1]*y₁ + s.q[2]*y₂)
    @constraint(model, 6*y₁ + 10*y₂ <= 60*x₁)
    @constraint(model, 8*y₁ + 5*y₂ <= 80*x₂)
end
```

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Creates a generator function for the first stage model

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    @constraint(model, 6*y₁ + 10*y₂ <= 60*x₁)
    @constraint(model, 8*y₁ + 5*y₂ <= 80*x₂)
end
```

Creates a generator function for the second stage model

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@first_stage sp = begin
    @variable(model, x₁ >= 40)
    @variable(model, x₂ >= 20)
    @objective(model, Min, 100*x₁ + 150*x₂)
    @constraint(model, x₁+x₂ <= 120)
end

@second_stage sp = begin
    @decision x₁ x₂
    s = scenario
    @variable(model, 0 <= y₁ <= s.d[1])
    @variable(model, 0 <= y₂ <= s.d[2])
    @objective(model, Min, s.q[1]*y₁ + s.q[2]*y₂)
    @constraint(model, 6*y₁ + 10*y₂ <= 60*x₁)
    @constraint(model, 8*y₁ + 5*y₂ <= 80*x₂)
end
```

Explicitly denote that some variables originate from the first stage

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    @objective(model, Min, s.q[1]*y₁ + s.q[2]*y₂)
    @constraint(model, 6*y₁ + 10*y₂ <= 60*x₁)
    @constraint(model, 8*y₁ + 5*y₂ <= 80*x₂)
end
```

Injection point for scenario data

StochasticPrograms.jl - Simple Example

```julia
struct SimpleScenario <: AbstractScenarioData
    p::Float64
    d::Vector{Float64}
    q::Vector{Float64}
end

StochasticPrograms.probability(s::SimpleScenario) = s.p
```

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Add two scenarios to the stochastic program

```julia
s1 = SimpleScenario(0.4,[500.0,100],[-24.0,-28])

s2 = SimpleScenario(0.6,[300.0,300],[-28.0,-32])

append!(sp,[s1,s2])
```

StochasticPrograms.jl - Simple Example

```
print(sp)
```

StochasticPrograms.jl - Simple Example

```
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First-stage
==============
Min 100 x₁ + 150 x₂
Subject to
x₁ + x₂ ≤ 120
x₁ ≥ 40
x₂ ≥ 20

Second-stage
==============
Subproblem 1:
Min -24 y₁ - 28 y₂
Subject to
6 y₁ + 10 y₂ - 60 x₁ ≤ 0
8 y₁ + 5 y₂ - 80 x₂ ≤ 0
0 ≤ y₁ ≤ 500
0 ≤ y₂ ≤ 100

Subproblem 2:
Min -28 y₁ - 32 y₂
Subject to
6 y₁ + 10 y₂ - 60 x₁ ≤ 0
8 y₁ + 5 y₂ - 80 x₂ ≤ 0
0 ≤ y₁ ≤ 300
0 ≤ y₂ ≤ 300
```

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Implications

- Flexible model creation and reformulation
- Efficient parallel implementation
- Versatility

StochasticPrograms.jl - Deterministically Equivalent Model

$$\begin{aligned} \underset{x \in \mathbb{R}^n}{\text{minimize}} \quad & c^T x + \mathbb{E}_\omega[Q(x, \xi(\omega))] \\ \text{s.t.} \quad & Ax = b \end{aligned}$$

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dep = DEP(sp)
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```
Minimization problem with:
* 5 linear constraints
* 6 variables
Solver is ClpMathProg
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- First stage generator

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- Second stage generator on all available scenarios

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Solver is ClpMathProg
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- First stage generator
- Second stage generator on all available scenarios
- Connections possible due to the `@decision` annotation
- DEP model is cached internally until new scenarios are added

StochasticPrograms.jl - Deterministically Equivalent Model

```
print(dep)
```

StochasticPrograms.jl - Deterministically Equivalent Model

```
print(dep)

Min 100 x₁ + 150 x₂ - 9.6 y₁_1 - 11.2 y₂_1 - 16.8 y₁_2 - 19.2 y₂_2
Subject to
x₁ + x₂ ≤ 120
6 y₁_1 + 10 y₂_1 - 60 x₁ ≤ 0
8 y₁_1 + 5 y₂_1 - 80 x₂ ≤ 0
6 y₁_2 + 10 y₂_2 - 60 x₁ ≤ 0
8 y₁_2 + 5 y₂_2 - 80 x₂ ≤ 0
x₁ ≥ 40
x₂ ≥ 20
0 ≤ y₁_1 ≤ 500
0 ≤ y₂_1 ≤ 100
0 ≤ y₁_2 ≤ 300
0 ≤ y₂_2 ≤ 300
```

StochasticPrograms.jl - Solving Models

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- Extended form

```
solve(sp,solver=ClpSolver())
:Optimal

getobjectivevalue(sp)
 -855.83
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- L-shaped

```
solve(sp,solver=LShapedSolver(:ls,ClpSolver()))

L-Shaped Gap  Time: 0:00:01 (4 iterations)
  Objective:       -855.8333333333358
  Gap:             2.1229209144670507e-15
  Number of cuts:  5
:Optimal
```

StochasticPrograms.jl - Solving Models

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solve(sp,solver=ClpSolver())
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  Number of cuts:  5
:Optimal
```

- Convenience function (Value of the recourse problem)

```
VRP(sp,solver=ClpSolver())
 -855.83
```

StochasticPrograms.jl - Wait-And-See Models

$$
\begin{aligned}
\underset{x \in \mathbb{R}^n}{\text{minimize}} \quad & c^T x + Q(x, \tilde{\xi}) \\
\text{s.t.} \quad & Ax = b \\
& x \geq 0
\end{aligned}
$$

for given $\tilde{\xi}$

StochasticPrograms.jl - Wait-And-See Models

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for given $\tilde{\xi}$

```
ws = WS(sp,s1)
Minimization problem with:
 * 3 linear constraints
 * 4 variables
Solver is ClpMathProg
```

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- First stage generator
- Second stage generator on the given scenario

StochasticPrograms.jl - Wait-And-See Models

```
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```

StochasticPrograms.jl - Wait-And-See Models

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print(ws)

Min 100 x₁ + 150 x₂ - 24 y₁ - 28 y₂
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 x₁ + x₂ ≤ 120
 6 y₁ + 10 y₂ - 60 x₁ ≤ 0
 8 y₁ + 5 y₂ - 80 x₂ ≤ 0
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 x₂ ≥ 20
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\begin{aligned}
\underset{x \in \mathbb{R}^n}{\text{minimize}} \quad & c^T x + Q(x, \bar{\xi}) \\
\text{s.t.} \quad & Ax = b \\
& x \geq 0
\end{aligned}
$$

where

$$
\bar{\xi} = \mathbb{E}_\omega[\xi(\omega)]
$$

StochasticPrograms.jl - Expected Value Problems

$$
\begin{aligned}
\underset{x \in \mathbb{R}^n}{\text{minimize}} \quad & c^T x + Q(x, \bar{\xi}) \\
\text{s.t.} \quad & Ax = b \\
& x \geq 0
\end{aligned}
$$

where

$$\bar{\xi} = \mathbb{E}_\omega[\xi(\omega)]$$

Must be possible to take expectation over scenarios

```julia
function expected(scenarios::Vector{SimpleScenario})
    return SimpleScenario(sum([s.p for s in scenarios]),
                          sum([s.p*s.d for s in scenarios]),
                          sum([s.p*s.q for s in scenarios]))
end
```

StochasticPrograms.jl - Expected Value Problems

```
evp = EVP(sp)
Minimization problem with:
 * 3 linear constraints
 * 4 variables
Solver is ClpMathProg

print(evp)
```

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Min 100 x₁ + 150 x₂ - 26.4 y₁ - 30.4 y₂
Subject to
 x₁ + x₂ ≤ 120
 6 y₁ + 10 y₂ - 60 x₁ ≤ 0
 8 y₁ + 5 y₂ - 80 x₂ ≤ 0
 x₁ ≥ 40
 x₂ ≥ 20
 0 ≤ y₁ ≤ 380
 0 ≤ y₂ ≤ 220
```

StochasticPrograms.jl - Decision Evaulation

$$c^T\hat{x} + \mathbb{E}_\omega[Q(\hat{x}, \xi(\omega))]$$

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```
x̂ = [50,50];
```

```
eval_decision(sp,x̂ ,solver=ClpSolver())
356.0
```

- Create first stage variables using generator

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- Create first stage variables using generator
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StochasticPrograms.jl - Decision Evaluation

$$c^T \hat{x} + \mathbb{E}_\omega[Q(\hat{x}, \xi(\omega))]$$

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- Again, linking handled through `@decision`

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- Create first stage variables using generator
- Fixate them to the given values
- Generate the second stage problems
- Again, linking handled through `@decision`
- Solve resulting JuMP model

StochasticPrograms.jl - Stochastic Measures

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EEV(sp,solver=ClpSolver())
-568.92
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- Expected value of perfect information (EVPI = VRP - EWS)

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- Value of the stochastic solution (VSS = EEV - VRP)

```
VSS(sp,solver=ClpSolver())
286.92
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StochasticPrograms.jl - Stochastic Measures

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Many of the required calculations are embarassingly parallel

LShapedSolvers.jl

L-shaped algorithm variants

- L-shaped [Van Slyke,Wets]
- Multicut L-shaped [Birge,Louveaux]
- Regularized decomposition [Ruszczyński]
- Trust-region L-shaped [Linderoth,Wright]
- Level-set [Fábián,Szőke]

LShapedSolvers.jl

- **L-shaped variants**
  1. L-shaped with multiple cuts (default): `LShapedSolver(:ls)`
  2. L-shaped with regularized decomposition: `LShapedSolver(:rd)`
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- Subproblems are solved using MathProgBase solvers

HydroModels.jl

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Creating a Planning Problem

- Define model indices
- Define model data
- Define `modelindices(::AbstractHydroModelData, ::Horizon, args...)`
- Define optimization problem

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Creating a Planning Problem

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Data injection keywords

- *horizon*: the time horizon if the model
- *indices*: structure with model indices
- *data*: structure with model data

HydroModels.jl - Simple Example

```julia
struct SimpleShortTermIndices <: AbstractModelIndices
    hours::Vector{Int}
    plants::Vector{Symbol}
end

struct SimpleShortTermData <: AbstractModelData
    hydrodata::HydroPlantCollection{Float64,2}
    D::Vector{Float64}   # Load balance
    λ::Vector{Float64}   # Price curve
end

function modelindices(data::SimpleShortTermData,horizon::Horizon)
    hours = collect(1:nhours(horizon))
    plants = data.hydrodata.plants
    if isempty(plants)
        error("No plants in data")
    end
    return SimpleShortTermIndices(hours, plants)
end
```

Define the required model indices

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Define data structure that should be available in the model

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end
```

Create model indices based on given data and time horizon

HydroModels.jl - Simple Example

```julia
@hydromodel Deterministic SimpleShortTerm = begin
    ...
    hours = indices.hours
    plants = indices.plants
    ...
    hdata = data.hydrodata
    D = data.D
    λ = data.λ
    ...
    @variable(model, H[t = hours] >= 0) # Production each hour
    ...
    @expression(model, value_of_stored_water,
                0.98*mean(λ)*sum(M[p,24]*sum(hdata[i].μ[1]
                    for i = hdata.Qd[p])
                        for p = plants))
    @objective(model, Max, net_profit + value_of_stored_water)
    ...
    @constraint(model, load_constraint[t = hours],
                H[t] + Hp[t] - Hs[t] == D[t])
    ...
end
```

HydroModels.jl - Simple Example

```
simple_model = SimpleShortTermModel(Day(),data)

Deterministic Hydro Power Model : Simple Short Term
    including 5 power stations
    over a 24 hour horizon (1 day)

Not yet planned
```

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plan!(simple_model, optimsolver = CbcSolver())

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Optimally planned
```

HydroModels.jl - Simple Example

```
reinitialize!(simple_model,Week(),data)

Deterministic Hydro Power Model : Simple Short Term
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  - 257 Swedish power stations
  - 20 Price curve scenarios from the NordPool market
  - 748042 variables and 376700 constraints in the extended form

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- HydroModels.jl model implemented using StochasticPrograms.jl
- Determine optimal order strategies on day-ahead electricity markets
- Small benchmark
  - 257 Swedish power stations
  - 20 Price curve scenarios from the NordPool market
  - 748042 variables and 376700 constraints in the extended form
- Results
  - Gurobi on extended form: 58.2 seconds (+ 9.2s for DEP generation)
  - Distributed L-shaped: 31.5 seconds
  - Distributed L-shaped with tuned trust-region: 26.7 seconds

Final Remarks - Outlook on Future Work

- StochasticPrograms.jl
  - Sampling
  - Multistage models
  - Progressive hedging solver

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- LShapedSolvers.jl
  - Algorithmic improvements
  - Hardware acceleration
  - Support integer problems

Final Remarks - Summary

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- Software innovations
  - Deferred model creation
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- Disclaimer: Not updated for MathOptInterface and JuMP 0.19
- All packages are available on Github:
  - https://github.com/martinbiel/StochasticPrograms.jl
  - https://github.com/martinbiel/LShapedSolvers.jl
  - https://github.com/martinbiel/HydroModels.jl

Feedback appreciated!