Sensor-based trajectory optimization

ABB Robotics

Master thesis
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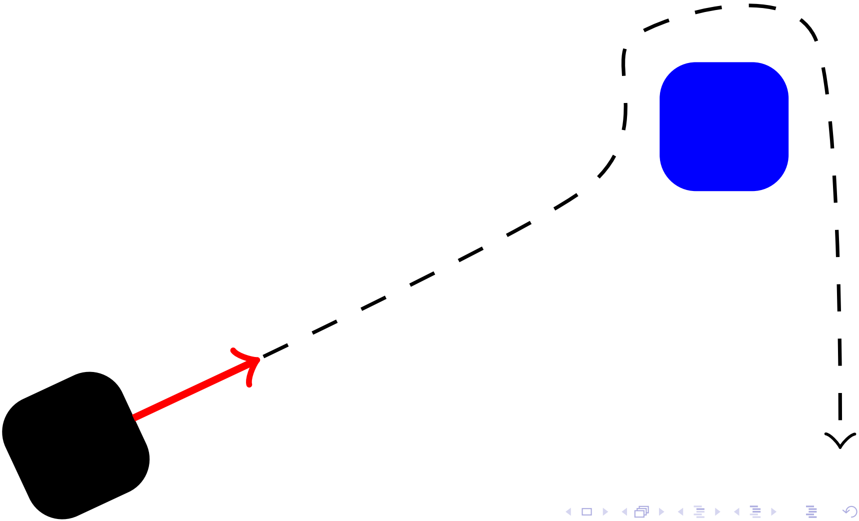
Examiner: Xiaoming Hu

June 9, 2016

Outline

Traditional approach

- Geometric path computed on before hand.



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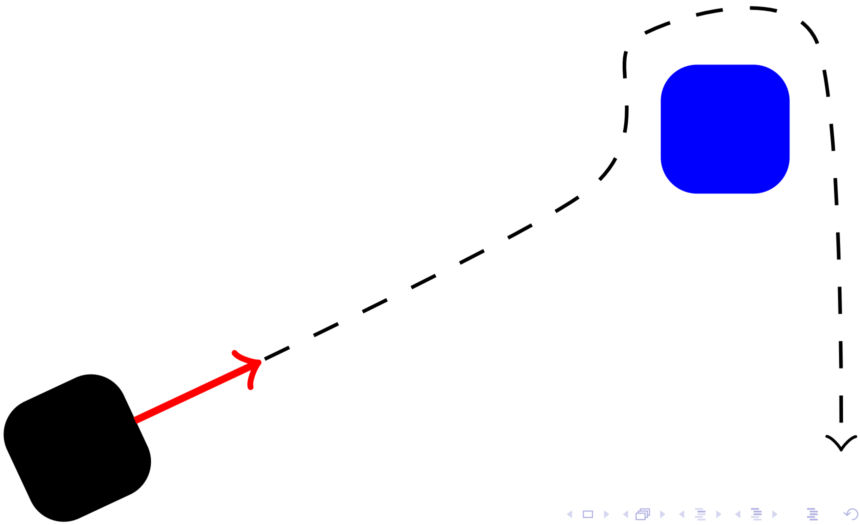

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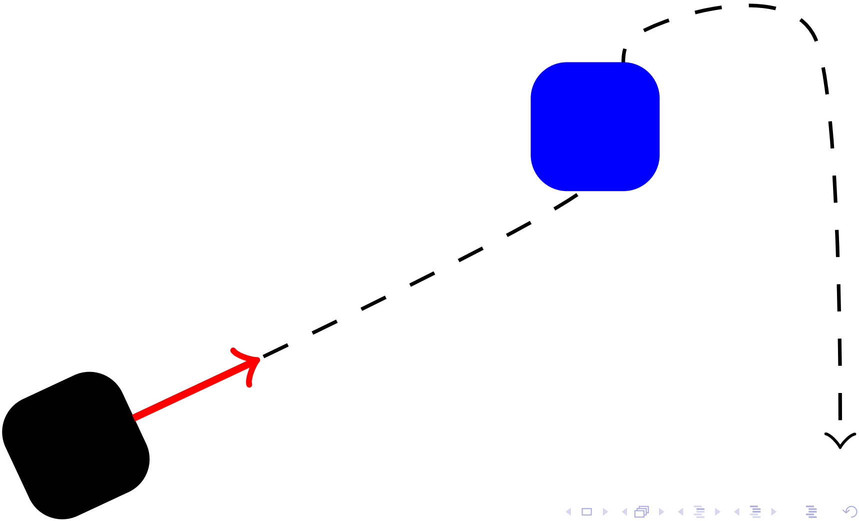

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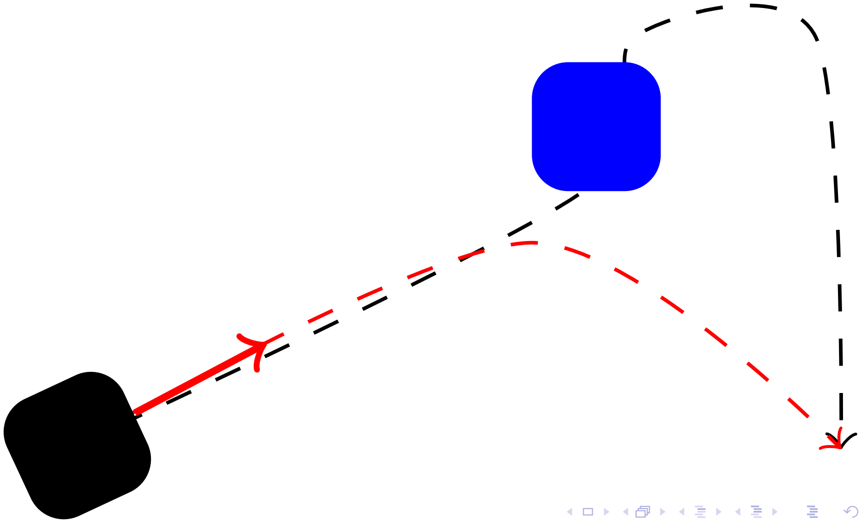

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- Track moving targets.

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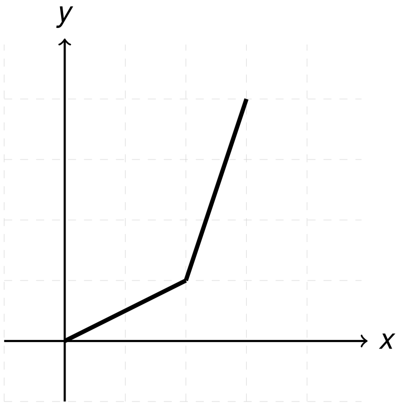

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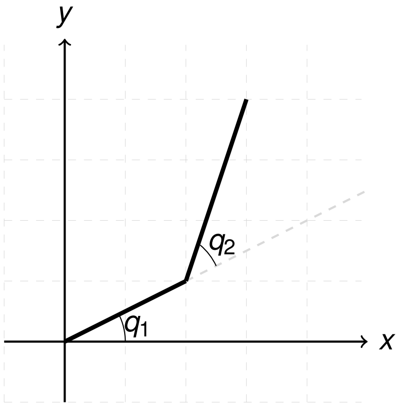

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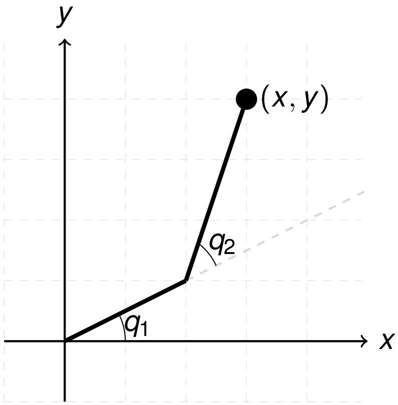

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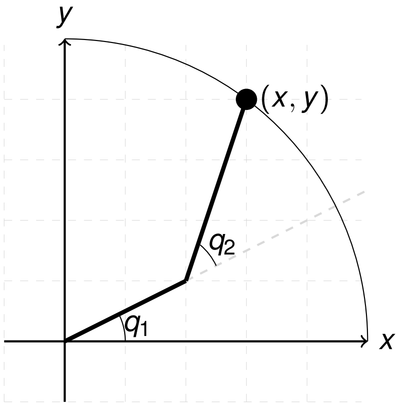

- $\mathcal{Q}$ - *Configuration space*
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- $\mathcal{W}$ - *Workspace*

Preliminaries - Robot modelling

- *Forward kinematics*: $\boldsymbol{y} = \chi_y(\boldsymbol{q})$
- *Inverse kinematics*: $\boldsymbol{q} = \chi_y^{-1}(\boldsymbol{y})$
- *Velocity Jacobian*: $\boldsymbol{v} = J(\boldsymbol{q})\dot{\boldsymbol{q}}$
- *Dynamics*: $M(\boldsymbol{q}(t))\ddot{\boldsymbol{q}}(t) + C(\boldsymbol{q}(t), \dot{\boldsymbol{q}}(t))\dot{\boldsymbol{q}}(t) + g(\boldsymbol{q}(t)) = \boldsymbol{\tau}(t)$

Preliminaries - Optimal control problem

Time minimizing formulation

$$\min_{\boldsymbol{\tau}(.)} T \quad \text{s.t.} \quad \begin{cases} M(\boldsymbol{q}(t))\ddot{\boldsymbol{q}}(t) + C(\boldsymbol{q}(t), \dot{\boldsymbol{q}}(t))\dot{\boldsymbol{q}}(t) + g(\boldsymbol{q}(t)) = \boldsymbol{\tau}(t) \\ \boldsymbol{q}(t) \in \mathcal{Q} \\ \boldsymbol{\tau}_- \leq \boldsymbol{\tau}(t) \leq \boldsymbol{\tau}_+ \\ \boldsymbol{y}(t) = \chi_y(\boldsymbol{q}(t)) \\ \boldsymbol{y}(0) = \boldsymbol{y}_0, \dot{\boldsymbol{y}}(0) = \dot{\boldsymbol{y}}_0 \\ \boldsymbol{y}(T) = \boldsymbol{y}_T, \dot{\boldsymbol{y}}(T) = \dot{\boldsymbol{y}}_T \end{cases}$$

Preliminaries - Timed elastic band

- Introduce the state vector

$$\boldsymbol{x}(t) = \begin{pmatrix} \boldsymbol{q}(t) \\ \dot{\boldsymbol{q}}(t) \end{pmatrix}$$

as a solution trajectory to the optimal control problem.

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- Discretize the trajectory into a so called *Timed Elastic Band* (TEB) set $\mathcal{B} := \{\boldsymbol{x}_1, \boldsymbol{\tau}_1, \boldsymbol{x}_2, \boldsymbol{\tau}_2, \dots, \boldsymbol{x}_{n-1}, \boldsymbol{\tau}_{n-1}, \boldsymbol{x}_n, \Delta T\}$. Note that n and ΔT are NOT fixed.

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- Determine the system dynamics for $\boldsymbol{x}(t)$ and approximate them using forward Euler,

$$\frac{\boldsymbol{x}_{k+1} - \boldsymbol{x}_k}{\Delta T} = A\boldsymbol{x}_k + B(f(\boldsymbol{x}_k) + h(\boldsymbol{x}_k)\boldsymbol{\tau}_k)$$

Preliminaries - Timed elastic band

$$
\begin{aligned}
\min_{\mathcal{B}} \quad & (n-1)\Delta T \\
\text{s.t.} \quad & \frac{\boldsymbol{x}_{k+1} - \boldsymbol{x}_k}{\Delta T} - A\boldsymbol{x}_k + B(f(\boldsymbol{x}_k) + h(\boldsymbol{x}_k)\boldsymbol{\tau}_k) = 0 \quad (k = 1, 2, \dots, n-1) \\
& \boldsymbol{\tau}_- \leq \boldsymbol{\tau}_k \leq \boldsymbol{\tau}_+ \quad (k = 1, 2, \dots, n-1) \\
& \boldsymbol{x}_1 = \boldsymbol{x}_s, \;\; \boldsymbol{x}_n = \boldsymbol{x}_f, \;\; \Delta T > 0
\end{aligned}
$$

$$
\left(\boldsymbol{x}_s = \begin{pmatrix} \boldsymbol{q}_s \\ \dot{\boldsymbol{q}}_s \end{pmatrix}, \quad \boldsymbol{x}_f = \begin{pmatrix} \chi_y^{-1}(\boldsymbol{y}_T) \\ \boldsymbol{0} \end{pmatrix} \right)
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The optimization problem is solved on-line using non-linear model predictive control techniques, in the timed elastic band framework.

Outline

Trajectory Planner - Deformation

Deformation in time

Deformation in space

Trajectory Planner - Deformation

Deformation in time

During each control cycle, the following TEB update is performed $\bar{I}_{TEB}$ times

$$\text{TEB update i}-\begin{cases} \text{Insert a new state if } \Delta T_i > \Delta \bar{T}_{ref} + \Delta \bar{T}_{hyst} \wedge n_i < \bar{n}_{max} \\ \text{Remove a state if } \Delta T_i < \Delta \bar{T}_{ref} - \Delta \bar{T}_{hyst} \wedge n_i > \bar{n}_{min} \\ \text{Leave the TEB unchanged otherwise} \end{cases}$$

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- The underlying solver is based on *Sequential Quadratic Programming* and employs line-search through an l_1 merit function.

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In total, $\bar{I}_{TEB} \cdot \bar{I}_{SQP}$ optimization iterations are performed each cycle.

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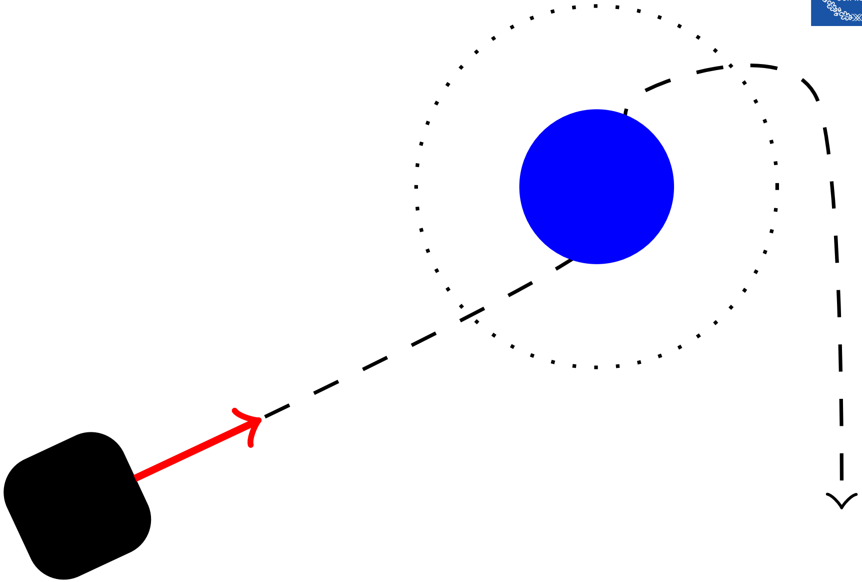
Trajectory Planner - Collision avoidance



Trajectory Planner - Collision avoidance

$$\min_{\mathcal{B}} \quad (n-1)\Delta T - \sum_{j=1}^{m} \sum_{k \in K_{j,\bar{\sigma}_{op}}} ||\chi_y(C\boldsymbol{x}_k) - \mathcal{O}_j||^2$$

$$\text{s.t.} \quad \frac{\boldsymbol{x}_{k+1} - \boldsymbol{x}_k}{\Delta T} - A\boldsymbol{x}_k + B(f(\boldsymbol{x}_k) + g(\boldsymbol{x}_k)\boldsymbol{\tau}_k) = 0 \quad (k = 1, \ldots, n-1)$$

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- Alternatively, the target state is replaced with the prediction

$$\boldsymbol{y}_i^{tg} + (n_{i-1} - 1)\Delta T_i \boldsymbol{v}$$

Outline

Algorithm 1 Trajectory Planning

Input: $\boldsymbol{q}_s$ - current state; $\dot{\boldsymbol{q}}_s$ - current velocity; $\boldsymbol{y}_f$ - target; $\dot{\boldsymbol{y}}_f$ - target velocity; $\mathcal{O}$ - obstacle information

Output: (Sub-)optimal control input $\boldsymbol{\tau}$

procedure PlanTrajectory
    repeat
        $(\boldsymbol{q}_s, \dot{\boldsymbol{q}}_s, \boldsymbol{y}_f, \dot{\boldsymbol{y}}_f) \leftarrow$ ReadSensorInput
        $\mathcal{O} \leftarrow$ InformAboutObstacles
        for each iteration 1 to $\bar{I}_{TEB}$ **do**
            $\mathcal{B} \leftarrow$ DeformInTime($\mathcal{B}$)
            $P \leftarrow$SetupUnderlyingProblem($\mathcal{B}, \mathcal{O}, \boldsymbol{q}_s, \dot{\boldsymbol{q}}_s, \boldsymbol{y}_f, \dot{\boldsymbol{y}}_f$)
            for each iteration 1 to $\bar{I}_{SQP}$ **do**
                $\mathcal{B} \leftarrow$ SQPSolve($\mathcal{B}$,P)
            end for
        end for
        $\tau \leftarrow$ ApplyControl($\mathcal{B}$)
    until target has been reached
end procedure

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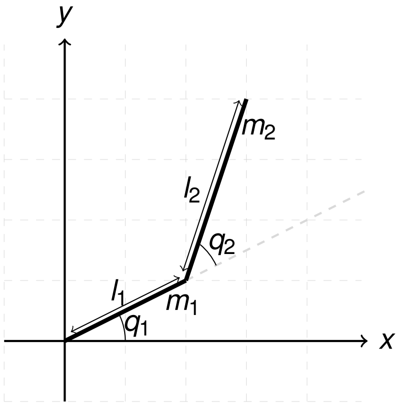
- In specific applications, the trajectory planner is extended in a subclass that configures the planner and provides the appropriate system dynamics.

Outline

Simulations - PlanarElbow/SCARA model



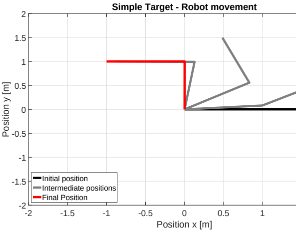
Outline

Scenario 1: Pick and place

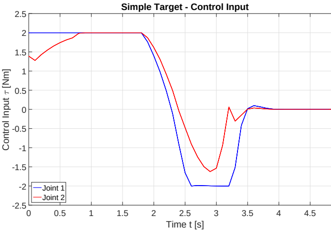
Demonstration

Scenario 1: Simple target - Time

(a) Snapshots of the intermediate planned trajectories, taken every 0.5s, together with the actual realized trajectory.



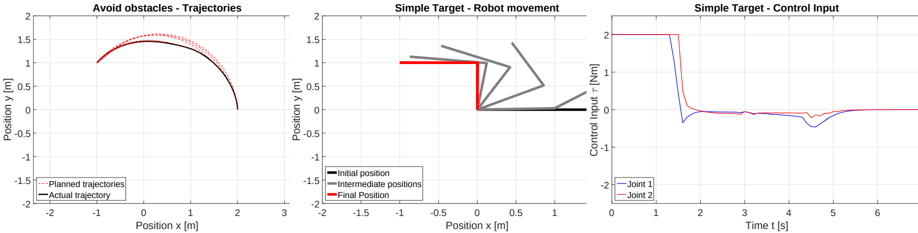
(b) The movement pattern of the robot



(c) The control input signal that was applied during the procedure.

Figure: Trajectory planning procedure for the *PlanarElbow* model with a simple stationary target at $(-1, 1)$ and aiming to minimize transition time. The planner was configured with the default values.

Scenario 1: Simple target - Energy



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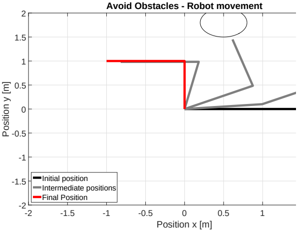
Outline

Scenario 2: Avoid obstacles

(a) Snapshots of the intermediate planned trajectories together with the actual realized trajectory.



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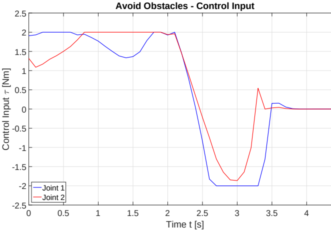

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Figure: A single obstacle with radius $0.3\mathrm{m}$ is placed at $(0.5, 1.8)$. The planner was configured with the default values.

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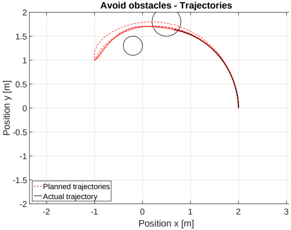

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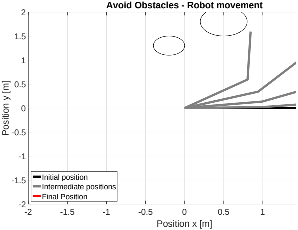

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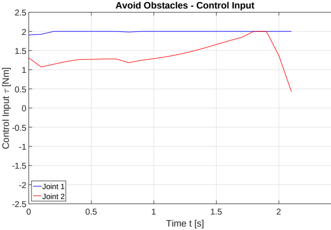

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Figure: Two obstacles are added to the workspace: one at $(0.5, 1.8)$ with radius $0.3\mathrm{m}$ and one at $(-0.2, 1, 3)$ with radius $0.2\mathrm{m}$. The planner was configured with the default values.

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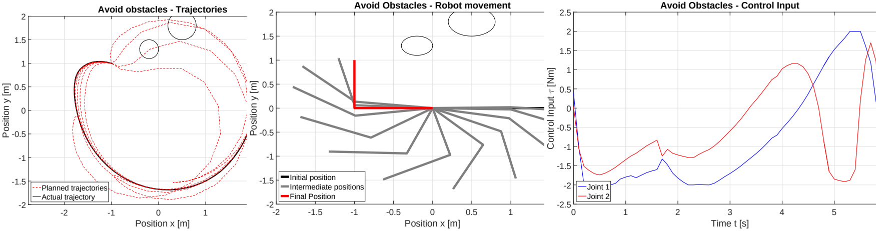

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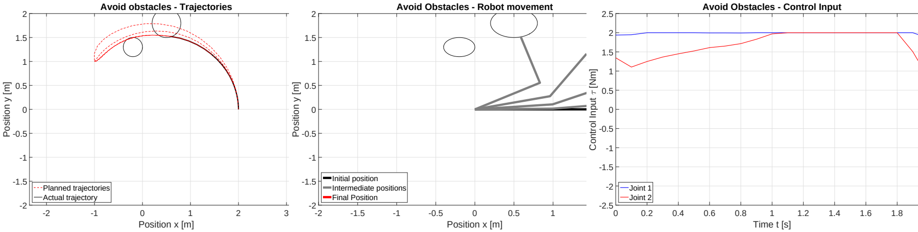

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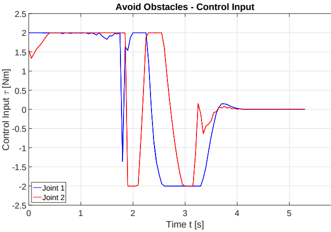

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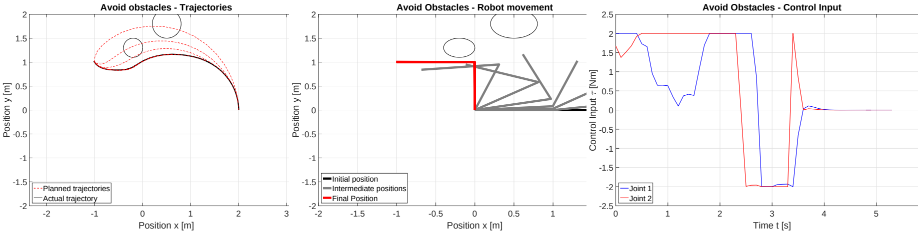

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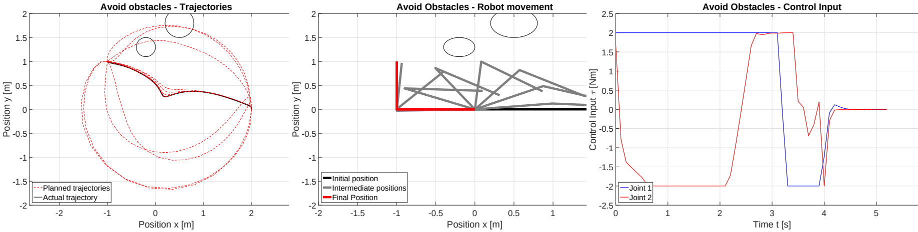
Figure: Two obstacles are added to the workspace: one at $(0.5, 1.8)$ with radius $0.3\mathrm{m}$ and one at $(-0.2, 1, 3)$ with radius $0.2\mathrm{m}$. The planner was configured with the default values, but with `"Isqp" : 4`.

Scenario 2: Avoid obstacles



(a) Snapshots of the intermediate planned trajectories together with the actual realized trajectory.

(b) The movement pattern of the robot.

(c) The control input signal that was applied during the procedure.

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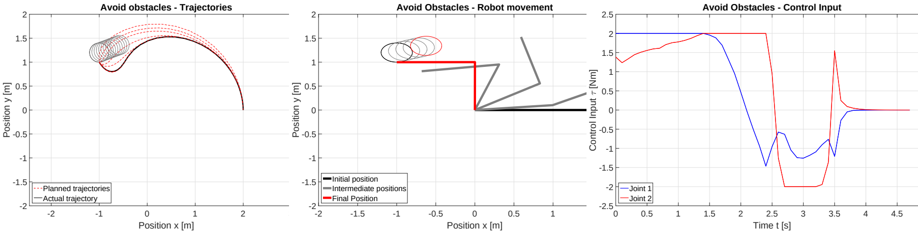


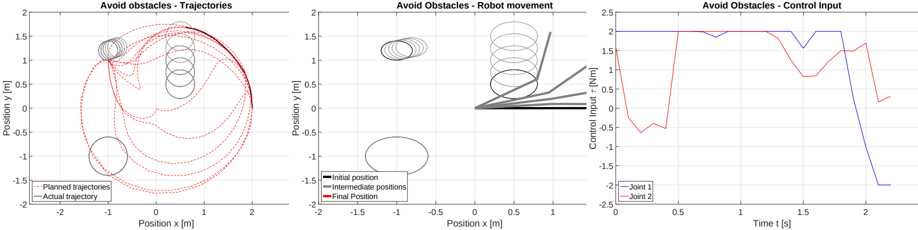
(a) Snapshots of the intermediate planned trajectories together with the actual realized trajectory.

(b) The movement pattern of the robot and the moving obstacle.

(c) The control input signal that was applied during the procedure.

Figure: A single obstacle with radius $0.2\mathrm{m}$ is placed at $(1, 1.2)$, and moving in the direction $(0.94, 0.35)$ with speed $0.1\mathrm{m/s}$. The planner was configured with the default values.

Scenario 2: Avoid obstacles



(a) Snapshots of the intermediate planned trajectories together with the actual realized trajectory.

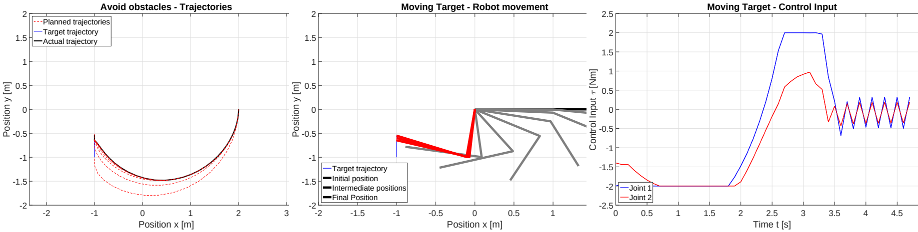
(b) The movement pattern of the robot and the moving obstacles.

(c) The control input signal that was applied during the procedure.

Figure: Three obstacles are added to the workspace: One at $(1, 1.2)$ with radius $0.2\mathrm{m}$, moving in the direction $(0.94, 0.35)$ with speed $0.1\mathrm{m}/\mathrm{s}$; One stationary obstacle at $(-1, -1)$ with radius $0.4\mathrm{m}$; and finally one at $(0.5, 0.5)$ with radius $0.3\mathrm{m}$, moving in the direction $(0, 1)$ with speed $5\mathrm{m}/\mathrm{s}$. The planner was configured with the default values, but with `"multipleTrajectories" : true`.

Outline

Scenario 3: Track moving target



(a) Snapshots of the intermediate planned trajectories together with the actual realized trajectory.

(b) The movement pattern of the robot and the moving target.

(c) The control input signal that was applied during the procedure.

Figure: Trajectory planning procedure for the *PlanarElbow* model with a moving target initially located at $(-1, -1)$ and moving in the direction $(0, 1)$ with speed $0.1\mathrm{m/s}$. The aim is to minimize transition time. The planner was configured with the default values.

Outline

Demonstration

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- More work required to make the planner real-time capable.

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- Robust MPC techniques to counter issues that arise from inaccurate models.

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- The intended application has been robotic manipulators, but the planner can be extended to other optimal control problems.
- The planner is successively applied in simple scenarios, but needs further work before it can be used in a real applications.

Questions?