



# Optimal Day-Ahead Orders Using Stochastic Programming and Noise-Driven Recurrent Neural Networks

Martin Biel

KTH - Royal Institute of Technology

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# Contribution



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## Motivation

- Determine optimal order strategies in a deregulated electricity market

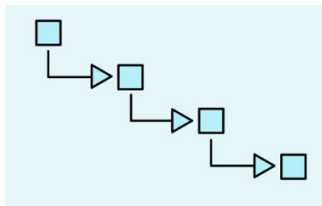
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- Hydroelectric power production
- Spatial dependence



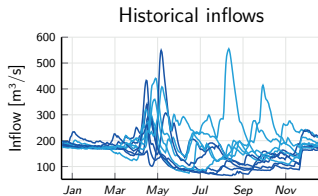
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- Spatial dependence
- Temporal dependence



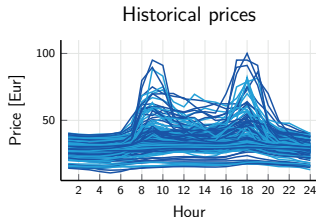
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- Determine optimal order strategies in a deregulated electricity market
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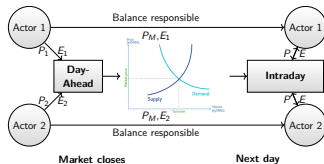
- Determine optimal order strategies in a deregulated electricity market
- Uncertain local inflow
- Uncertain electricity price





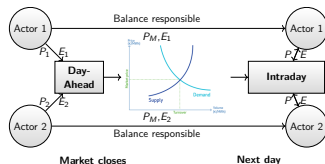
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- Uncertain electricity price
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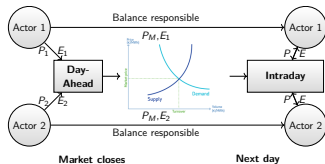


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- Complete modeling procedure
  - ▶ Data gathering
  - ▶ Forecast generation
  - ▶ Model formulation
  - ▶ Optimization
  - ▶ Result visualization

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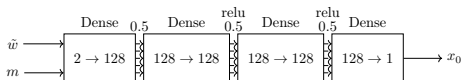
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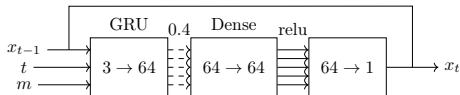
Martin Biel. [Optimal day-ahead orders using stochastic programming and noise-driven RNNs.](#)

*arXiv preprint arXiv:1910.04510, 2019*

- Noise-driven recurrent neural network

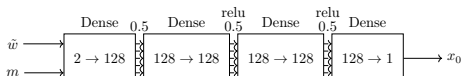


**Figure:** Initializer network in the price forecaster.

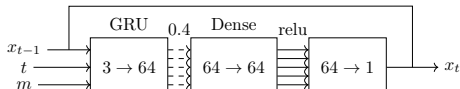


**Figure:** Sequence generation network in the price forecaster.

- Noise-driven recurrent neural network
- Trained on price data and inflow data separately



**Figure:** Initializer network in the price forecaster.



**Figure:** Sequence generation network in the price forecaster.

- Noise-driven recurrent neural network
- Trained on price data and inflow data separately
- Seasonality modeled through separate inputs to the network

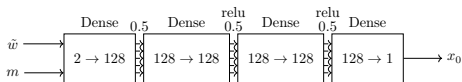


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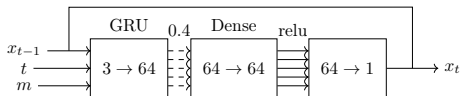
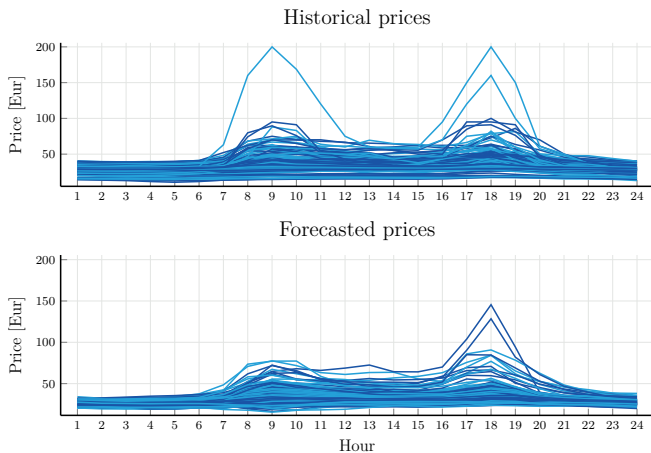


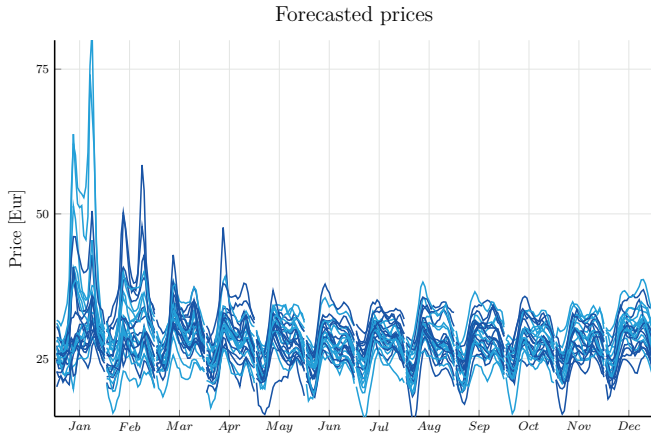
Figure: Sequence generation network in the price forecaster.

# Forecast generation



**Figure:** Historical electricity price curves in January and electricity price curves generated using the RNN forecaster in the same period.

# Forecast generation



**Figure:** Daily electricity price curves predicted by the RNN forecaster in every month of the year.





# Model formulation

- `StochasticPrograms.jl`: framework for stochastic programming



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- `StochasticPrograms.jl`: framework for stochastic programming
- Formulate, solve and analyze stochastic models
- Distributed-memory implementation for large-scale models
- Efficient implementations of structure-exploiting algorithms

# Model formulation - StochasticPrograms.jl

```
@stochastic_model simple begin
  @stage 1 begin
    @decision(simple, x1 >= 40)
    @decision(simple, x2 >= 20)
    @objective(simple, Min, 100*x1 + 150*x2)
    @constraint(simple, x1 + x2 <= 120)
  end
  @stage 2 begin
    @uncertain q1 q2 d1 d2
    @recourse(simple, 0 <= y1 <= d1)
    @recourse(simple, 0 <= y2 <= d2)
    @objective(simple, Max, q1*y1 + q2*y2)
    @constraint(simple, 6*y1 + 10*y2 <= 60*x1)
    @constraint(simple, 8*y1 + 5*y2 <= 80*x2)
  end
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end
```

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$$\begin{aligned} & \underset{x_1, x_2 \in \mathbb{R}}{\text{minimize}} && 100x_1 + 150x_2 \\ & \text{subject to} && x_1 + x_2 \leq 120 \\ & && x_1 \geq 40 \\ & && x_2 \geq 20 \end{aligned}$$

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```

$$\begin{aligned}
 & \max_{y_1, y_2 \in \mathbb{R}} q_1(\xi) y_1 + q_2(\xi) y_2 \\
 & \text{s.t. } 6y_1 + 10y_2 \leq 60 x_1 \\
 & \quad 8y_1 + 5y_2 \leq 80 x_2 \\
 & \quad 0 \leq y_1 \leq d_1(\xi) \\
 & \quad 0 \leq y_2 \leq d_2(\xi)
 \end{aligned}$$



# Model formulation - Day-ahead problem

Full model details:

- Preprint: Martin Biel. [Optimal day-ahead orders using stochastic programming and noise-driven RNNs](#).  
*arXiv preprint arXiv:1910.04510*, 2019
- Github: [github.com/martinbiel/HydroModels](https://github.com/martinbiel/HydroModels)





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- Ensure a statistically significant *value of the stochastic solution*



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- Sampled instances of  $\sim 2000$  scenarios required to reach this bound
  - ▶  $\sim 5$  million variables
  - ▶  $\sim 3.3$  million constraints



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- Ensure a statistically significant *value of the stochastic solution*
- Sampled instances of  $\sim 2000$  scenarios required to reach this bound
  - ▶  $\sim 5$  million variables
  - ▶  $\sim 3.3$  million constraints
- Leverage distributed capabilities of `StochasticPrograms.jl`

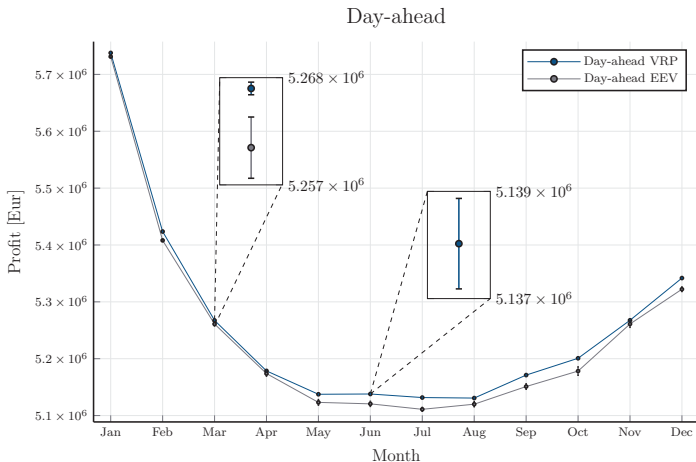


Figure: Seasonal variation of day-ahead VRP and EEV, including 95% confidence intervals.



# Final remarks

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- Large-scale day-ahead problems solved on a compute cluster



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- Proof of concept for large-scale models in `StochasticPrograms.jl`



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- Noise-driven recurrent neural networks to sample scenarios
- Tight confidence intervals from SAA
- Statistically significant VSS
- Proof of concept for large-scale models in `StochasticPrograms.jl`

## Further information

- Contact: `mbiel@kth.se`
- Github: `github.com/martinbiel/StochasticPrograms.jl`
- Full paper